

# Adaptive Monte Carlo methods

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where

- $\pi$  is a probability distribution defined on a space  $\mathcal{X} \subset \mathbb{R}^{n_x}$ ,
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Calculating  $I(f)$  analytically might be impossible: one resorts to numerical approximations

- Exploit the law(s) of large numbers to estimate  $\mathbb{E}_\pi(f)$  with *iid* samples from  $\pi$  with

$$\hat{I}_N(f) = \frac{1}{N} \sum_{i=1}^N f(X_i)$$

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- It is rarely the case that such *iid* samples can be obtained in practice,
- One resorts to iterative methods (Sequential Monte Carlo methods, Markov chain Monte Carlo methods) which depend on tuning parameters.

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- It consists of constructing an *ergodic* Markov chain (MC)  $\{X_i\}$  ( $i = 1, 2, \dots$ ) with *invariant* distribution  $\pi$ .
- And compute the estimator

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- 1  $X_{i+1} = y$  with probability  $\alpha(x, y)$
- 2 Otherwise,  $X_{i+1} = x$ .

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- The choice of  $q$  is key to the success of the MCMC approach.
- For example if

$$q_{\theta}(x, y) = \frac{1}{\sqrt{2\pi\theta^2}} \exp\left(\frac{-1}{2\theta^2} (y - x)^2\right).$$

the variance of  $\hat{I}_N(f)$  is large for values of  $\theta^2$  that are either too small or too large.

- Sample initial values  $X_0, \theta_0 \in \Theta \times X$ .
- Iteration  $i + 1$ , given  $\theta_i = \theta_i(X_0, \dots, X_i)$  and  $X_i$  from the previous iteration,
  - 1 Sample  $X_{i+1} | (X_0, \dots, X_i) \sim P_{\theta_i}(X_i, \cdot)$ ,
  - 2 Compute  $\theta_{i+1} = \theta_{i+1}(X_0, \dots, X_{i+1})$ .

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- Criteria guiding the choice of the updates  $\theta_i$ ?
- Framework to “optimise” such criteria.

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- Obviously, with  $\pi = (1/2 \ 1/2)$ ,

$$\pi P_\theta = \pi$$

i.e.  $\pi$  invariant distribution

- and converges if  $\theta \in \Theta = (0, 1)$ .

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- This still defines a time homogeneous MC with

$$\tilde{P}(X_{i+1} = b | X_i = a) = P_{\theta(a)}(X_{i+1} = b | X_i = a)$$

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- After some algebra... the invariant distribution is now

$$\tilde{\pi} = \left( \frac{1 - \theta(2)}{2 - \theta(1) - \theta(2)}, \frac{1 - \theta(1)}{2 - \theta(1) - \theta(2)} \right) \neq \pi .$$



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- There is extensive literature which establishes that this is indeed the case under reasonable conditions.

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- Let  $\tau(\theta)$  be the acceptance rate of the algorithm at stationarity

$$\tau(\theta) := \iint_{\mathcal{X} \times \mathcal{X}} \pi(x) \left( 1 \wedge \frac{\pi(y)}{\pi(x)} \right) q_\theta(x, y) dx dy.$$

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- Relevant theory says that it makes sense to choose  $\theta^*$  such that  $\tau(\theta^*) \approx \tau^* = 0.234$ .
- But in general  $\theta^*$  is not known. Therefore it is of interest to have an algorithm that automatically learns  $\theta^*$  by monitoring the acceptance rate of the algorithm in the long-run.

- Objective: find  $\theta$  that solves the equation

$$h(\theta) = \iint_{\mathcal{X} \times \mathcal{X}} \alpha(x, y) q_{\theta}(x, y) \pi(x) dx dy - \tau^* = 0 ,$$

here  $\alpha(x, y) = 1 \wedge \pi(y) / \pi(x)$ .



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- Suggestion :

$$Y_{k+1} \sim q_{\theta_k}(X_k, \cdot)$$

$$X_{k+1} \sim \begin{cases} Y_{k+1} & \text{with probability } \alpha(X_k, Y_{k+1}) \\ X_k & \text{otherwise} \end{cases}$$

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- Implicit assumption about monotonicity of  $\tau(\theta)$ .

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  - $\lambda = 2.38^2/n_x$ .
  - $\Gamma_\pi$  is the covariance matrix of  $\pi$ , unknown *a priori*!



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- 2 Set  $\gamma_{k+1} = 1/(k + 1)$  and update  $\mu_k, \Gamma_k$

$$\begin{aligned}\mu_{k+1} &= (1 - \gamma_{k+1})\mu_k + \gamma_{k+1}X_{k+1} \\ &= \mu_k + \gamma_{k+1}(X_{k+1} - \mu_k)\end{aligned}$$

One can rewrite the update for  $(\mu_{k+1}, \Gamma_{k+1})$  as follows,

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with  $\theta_{k+1} := (\mu_{k+1}, \Gamma_{k+1})$

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# Improving on the AM algorithm...

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- However in practice, especially if  $\Gamma_i$  is far from  $\Gamma_\pi$  (say very small)  $\lambda^*$  is likely to be inappropriate.
- It is therefore natural to combine the estimation of these quantities.

# AM algorithm with adaptive scaling

- 1 Given  $(\mu_i, \Gamma_i)$ , sample  $Y_{i+1} \sim \mathcal{N}(X_i; \mu_i, \exp(\lambda_i) \times \Gamma_i)$  and set  $X_{i+1} = Y_{i+1}$  with probability  $\alpha(X_i, Y_{i+1})$ , otherwise  $X_{i+1} = X_i$ .
- 2 Update

$$\log(\lambda_{i+1}) = \log(\lambda_i) + \gamma_{i+1}[\alpha(X_i, Y_{i+1}) - \alpha_*]$$

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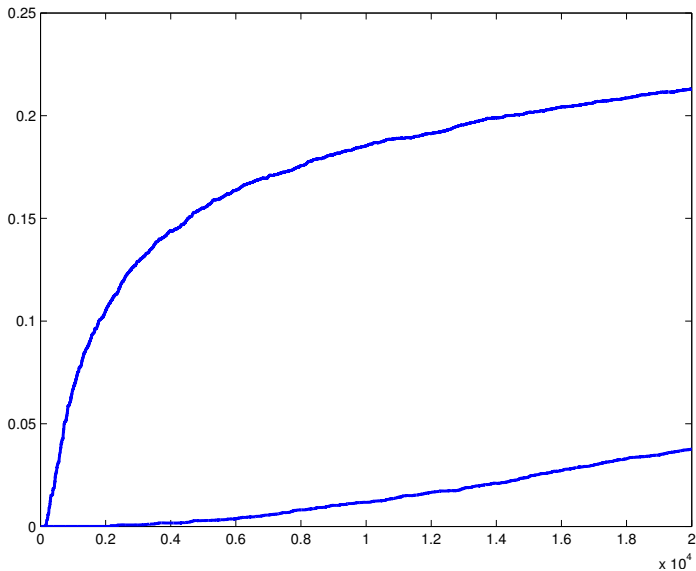
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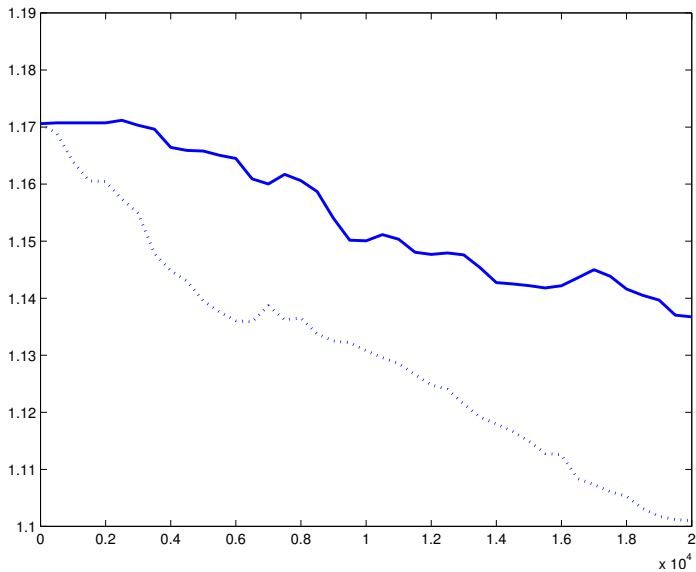
$$\Gamma_{i+1} = \Gamma_i + \gamma_{i+1}[(X_{i+1} - \mu_i)(X_{i+1} - \mu_i)^T - \Gamma_i] .$$

There are many possible variations on this theme which can significantly improve performance [Andrieu & Thoms, 2008]...

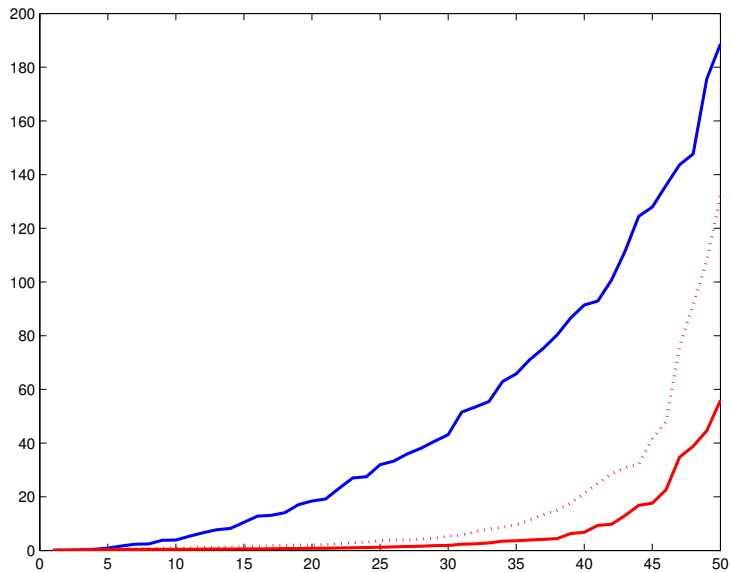
# A 50 dimensional target distribution



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  - in the ABC context or when using composite likelihoods in a Bayesian framework optimising the target distribution is required and there is a need for automation.



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- 5 PDRAs to be recruited during 2013,
- each position will be for 2 years (with opportunity for extension to 4 years),
- the positions are to be held at one of the four universities involved in the project,
- **IMPORTANT:** we encourage you to apply through the **FOUR** Universities.